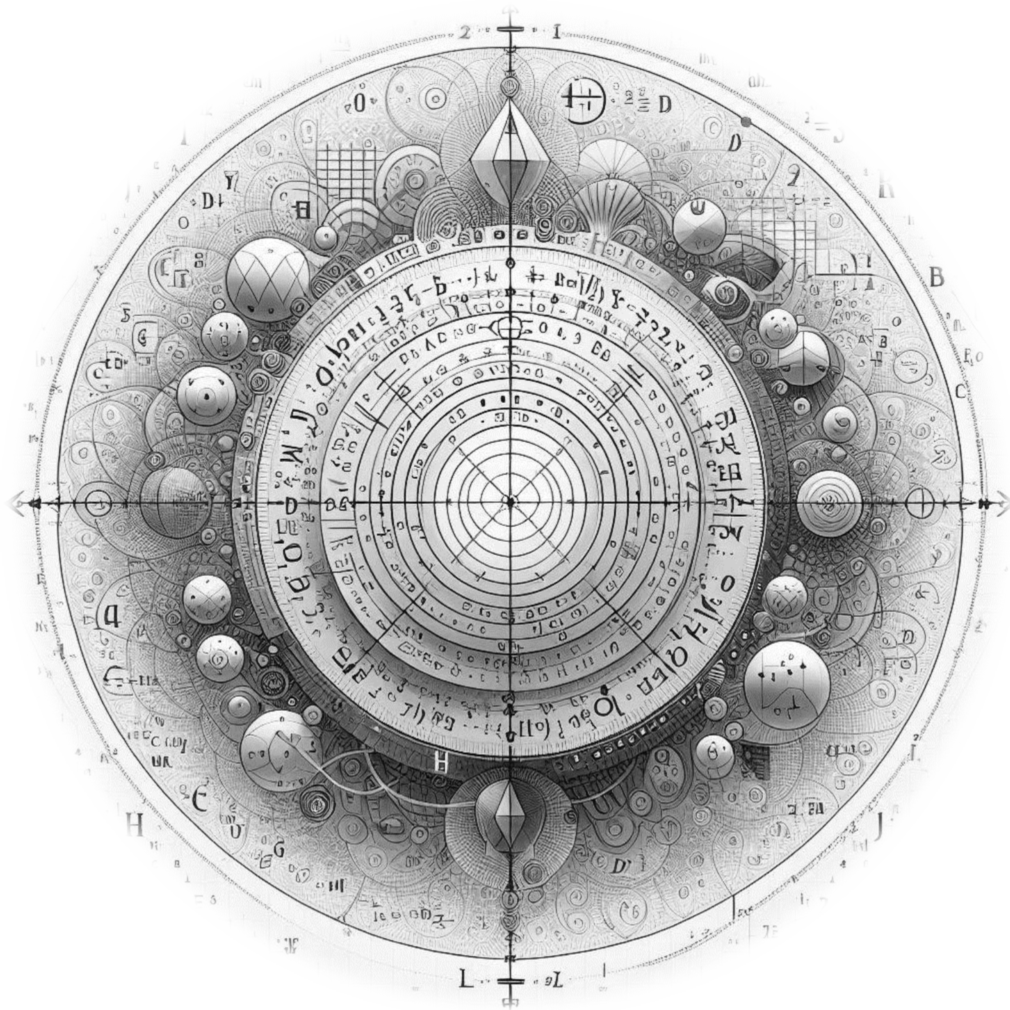
eSpyMath: Grade 7 Math Workbook

Contents

Chapter 1. Mastering Numbers: From Integers to Proportions



1-1. Understanding and applying integers and rational numbers in the number system

1. $4+(8\div 2)$ 2. $15-(3\times 5)$ 3. $(4+6)\times 2$ 4. $(10-5)\div 5$	8 0 20 1

1. Integers:
- Definition: Integers are whole numbers that can be positive, negative, or zero. They do not have fractional or decimal parts.
- Examples: -3, 0, 7.

2. Rational Numbers:
- Definition: Rational numbers are numbers that can be expressed as the quotient or fraction of two integers, where the denominator is not zero.
- Examples: 1/2, -3/4, 5.

3. Classifying Numbers:
- Integer: Any whole number, positive or negative, including zero.
- Rational Number: Any number that can be written as a fraction, which includes integers (since they can be written as fractions with a denominator of 1).

4. Converting Fractions to Decimals:
- A fraction can be converted to a decimal by dividing the numerator by the denominator.
- Example: 3/4 = 0.75.

5. Opposites and Absolute Values:
- Opposite: The opposite of an integer is the same number with the opposite sign.
- Absolute Value: The distance of a number from zero on the number line, regardless of direction.

Formulas

1. Adding and Subtracting Rational Numbers:
- Same Denominator: Add or subtract the numerators and keep the denominator.
- Different Denominators: Find a common denominator, convert, then add or subtract.

$$\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$$
$$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$$

2. Multiplying Rational Numbers:

- Multiply the numerators together and the denominators together.

$$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$$

3. Solving Linear Equations:

- To solve $3x+4=13$, isolate x.

$$3x+4=13\Rightarrow 3x=9\Rightarrow x=3$$

4. Finding the Whole Number from a Fraction:

- If $\frac{1}{2}$ of a number is 5, the whole number is found by multiplying by 2.

$$\frac{1}{2}x=5\Rightarrow x=10$$

Exercise

1) What is an integer? Give 3 examples of integers.	2) What is a rational number? Provide 2 examples.
3) Identify whether the number -8 is an integer, rational number, or both.	4) Convert the fraction 3/4 into a decimal.
5) Is the number 0.5 an integer, a rational number, or both? Explain.	6) What is the opposite of the integer -7?

7) Add the rational numbers 3/4 and 2/3 and simplify if possible.	8) What is the product of -2 and 5/6?
9) Solve for x in the equation 3x + 4 = 13.	10) If 1/2 of a number is 5, what is the whole number?

Solutions:

1) What is an integer? Give 3 examples of integers. - An integer is a whole number that can be either positive, negative, or zero. - Examples of integers include -5, 0, and 7.	**2) What is a rational number? Provide 2 examples.** - A rational number is any number that can be expressed as the quotient or fraction of two integers, where the denominator is not zero. - Examples include 1/2 and -3/4.
3) Identify whether the number -8 is an integer, rational number, or both. - The number -8 is both an integer and a rational number. - It is an integer because it is a whole number, and it is rational because it can be written as -8/1, which is the quotient of two integers.	**4) Convert the fraction 3/4 into a decimal.** - To convert the fraction 3/4 into a decimal, divide the numerator (3) by the denominator (4). - 3 ÷ 4 = 0.75.
5) Is the number 0.5 an integer, a rational number, or both? Explain. - The number 0.5 is not an integer because it is not a whole number. - However, it is a rational number because it can be expressed as the fraction 1/2, which is the quotient of two integers.	**6) What is the opposite of the integer -7?** - The opposite of the integer -7 is 7. - The opposite of an integer is the number that is the same distance from zero on the number line but in the opposite direction.
7) Add the rational numbers 3/4 and 2/3 and simplify if possible. - To add 3/4 and 2/3, first find a common denominator, which is 12 in this case. - Then, convert each fraction to have the common denominator: 3/4 = 9/12 and 2/3 = 8/12. - Add them together: 9/12 + 8/12 = 17/12 or $1\frac{5}{12}$.	**8) What is the product of -2 and 5/6?** - The product of -2 and 5/6 is found by multiplying the two numbers: (-2) × (5/6) = -10/6, which can be simplified to -5/3.
9) Solve for x in the equation 3x + 4 = 13. - To solve for x, first subtract 4 from both sides of the equation: 3x + 4 - 4 = 13 - 4. - This simplifies to 3x = 9. Then, divide both sides by 3: 3x/3 = 9/3. Thus, x = 3.	**10) If 1/2 of a number is 5, what is the whole number?** - If 1/2 of a number is 5, multiply both sides of the equation by 2 to find the whole number: 1/2 × number = 5, so number = 5 × 2. Therefore, the whole number is 10.

1-2. Operations with rational numbers

1. $(3+7)-4$ 2. $(8\times 2)\div 4$ 3. $10-(2+3)\times 2$ 4. $(12\div 6)+9$	6 4 0 11

1. Addition and Subtraction of Rational Numbers:

- To add or subtract fractions, they must have a common denominator. Convert fractions to equivalent fractions with the same denominator before performing the operation.

2. Multiplication of Rational Numbers:

- Multiply the numerators together and the denominators together. Simplify the result if possible.

3. Division of Rational Numbers:

- To divide by a fraction, multiply by its reciprocal (flip the numerator and denominator of the divisor).

4. Reciprocal of a Number:

- The reciprocal of a number $\frac{a}{b}$ is $\frac{b}{a}$.

5. Solving Equations Involving Fractions:

- To solve equations involving fractions, clear the fractions by multiplying through by the least common denominator (LCD) or by isolating the variable.

6. Understanding Sums and Products of Rational Numbers:

- The sum or product of two rational numbers can be determined by setting up equations based on the given conditions and solving for the unknowns.

Formulas

1. Addition/Subtraction of Rational Numbers:

- Same Denominator: $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$
- Different Denominators: $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$

2. Multiplication of Rational Numbers:

$$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$$

3. Division of Rational Numbers:

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$$

4. Reciprocal of a Rational Number:

$$\left(\frac{a}{b}\right)^{-1}=\frac{b}{a}$$

5. Solving Rational Equations:

- Clear fractions by multiplying by the LCD or isolate the variable:

$$\frac{5}{6x}-\frac{1}{2}=\frac{1}{3}$$

6. Sum and Product of Two Numbers:

- For sum S and product P :

$$x+y=S \text{ and } xy=P$$

Exercise

1) Add the rational numbers 1/4 and 1/3. Simplify your answer if possible.	2) Subtract 2/5 from 3/4. Give your answer as a simplified fraction.
3) Multiply the rational numbers -3/4 and 2/3. Simplify the result if necessary.	4) Divide the fraction 3/5 by the fraction 2/3. Give your answer as a simplified fraction.
5) What is the sum of -7/8 and 5/6? Simplify your answer.	6) If you subtract a number from its reciprocal, the result is 3/2. If the original number is a/b, write the equation representing this situation.
7) Solve the equation for x: (5/6)x - 1/2 = 1/3.	8) A recipe calls for 3/4 cup of sugar, but you accidentally pour in 1 cup. How much sugar must you remove to have the correct amount?
9) If the product of two rational numbers is -1 and one of the numbers is 4/5, what is the other number?	10) Two numbers have a sum of 5 and a product of 6. If one of the numbers is 2, find the other number.

Solutions:

1) Add the rational numbers 1/4 and 1/3. Simplify your answer if possible. - To add 1/4 and 1/3, find a common denominator, which is 12. - Then, convert each fraction: 1/4 = 3/12 and 1/3 = 4/12. - Add them together: 3/12 + 4/12 = 7/12.	**2) Subtract 2/5 from 3/4. Give your answer as a simplified fraction.** - To subtract 2/5 from 3/4, find a common denominator, which is 20. - Convert each fraction: 3/4 = 15/20 and 2/5 = 8/20. - Then subtract: 15/20 - 8/20 = 7/20.
3) Multiply the rational numbers -3/4 and 2/3. Simplify the result if necessary. - To multiply -3/4 by 2/3, multiply the numerators together and the denominators together: (-3 × 2) / (4 × 3) = -6/12, which simplifies to -1/2.	**4) Divide the fraction 3/5 by the fraction 2/3. Give your answer as a simplified fraction.** - To divide 3/5 by 2/3, multiply the first fraction by the reciprocal of the second fraction: 3/5 × 3/2 = 9/10.
5) What is the sum of -7/8 and 5/6? Simplify your answer. - Find a common denominator, which is 24. - Convert each fraction: -7/8 = -21/24 and 5/6 = 20/24. Then add: -21/24 + 20/24 = -1/24.	**6) If you subtract a number from its reciprocal, the result is 3/2. If the original number is a/b, write the equation representing this situation.** - The equation is a/b - b/a = 3/2.
7) Solve the equation for x: (5/6)x - 1/2 = 1/3. - First, find a common denominator for the fractions, which is 6, and rewrite the equation: (5/6)x - 3/6 = 2/6. - Then, add 3/6 to both sides: (5/6)x = 5/6. - Dividing both sides by 5/6 gives x = 1.	**8) A recipe calls for 3/4 cup of sugar, but you accidentally pour in 1 cup. How much sugar must you remove to have the correct amount?** - Subtract the correct amount from what you added: 1 cup - 3/4 cup = 1/4 cup. - You must remove 1/4 cup of sugar.
9) If the product of two rational numbers is -1 and one of the numbers is 4/5, what is the other number? - To find the other number, divide -1 by 4/5: -1 ÷ (4/5) = -1 × (5/4) = -5/4. - The other number is -5/4.	**10) Two numbers have a sum of 5 and a product of 6. If one of the numbers is 2, find the other number.** - Let the two numbers be x and y . - Given: x + y = 5 and $x \times y = 6$. Identify the known number: - One of the numbers is given as 2 . - Let's say $x = 2$. Substitute x in the sum equation: - Since x = 2 , substitute x in $x + y = 5$: $2 + y = 5$ Solve for y : - Subtract 2 from both sides of the equation: $y = 5 - 2$ - Simplify: $y = 3$ - Therefore, the other number is 3 .

1-3. Real-life applications of rational number operations

1. $7\times(3-1)$ 2. $(9+3)\div 3$ 3. $8-(4\div 2)\times 3$ 4. $(15-5)\times(2+1)$	14 4 2 30

1. Doubling a Quantity:

- To double a quantity, multiply it by 2.

2. Dividing a Length into Equal Parts:

- To find how many equal parts a length can be divided into, divide the total length by the length of each part.

3. Calculating Distance Based on Fuel Efficiency:

- To calculate how far you can drive, multiply the fuel efficiency (miles per gallon) by the amount of fuel left (gallons).

4. Multiplying Quantities and Costs:

- To find the total cost, multiply the quantity by the cost per unit.

5. Subtracting Quantities:

- To find how much more was added than required, subtract the required amount from the added amount.

6. Adding to a Capacity:

- To find how much more can be added to a tank, subtract the current amount from the total capacity.

7. Calculating Unpainted Fraction:

- To find the remaining fraction, subtract the painted fraction from the whole (1).

8. Finding Fraction of Games Lost:

- To find the fraction of games lost, subtract the fraction of games won from the total (1).

9. Increasing Daily Intake:

- To find the new daily intake, add the additional intake to the original intake.

10. Time to Complete a Phase:

- To find the time to complete a phase at a given rate, divide the total time by the fraction completed per unit of time.

Formulas:

1. Doubling a Quantity:

$$2 \times \text{quantity}$$

2. Dividing Length:

$$\frac{\text{Total Length}}{\text{Length of Each Part}}$$

3. Distance Based on Fuel Efficiency:

$$\text{Fuel Efficiency (miles per gallon)} \times \text{Fuel Left (gallons)}$$

4. Total Cost:

$$\text{Quantity} \times \text{Cost per Unit}$$

5. Difference in Quantities:

$$\text{Added Quantity} - \text{Required Quantity}$$

6. Additional Capacity:

$$\text{Total Capacity} - \text{Current Amount}$$

7. Remaining Fraction:

$$1 - \text{Painted Fraction}$$

8. Fraction of Games Lost:

$$1 - \text{Fraction of Games Won}$$

9. New Daily Intake:

$$\text{Original Intake} + \text{Additional Intake}$$

10. Time to Complete Phase:

$$\frac{\text{Total Time}}{\text{Fraction Completed per Unit Time}}$$

Exercise

1) If a recipe calls for 2/3 cup of milk and you want to double the recipe, how much milk will you need?	2) You have a rope that is $7\frac{1}{2}$ feet long. You need to cut it into pieces that are 3/4 foot long each. How many pieces can you cut?
3) A car's fuel efficiency is $20\frac{1}{2}$ miles per gallon. If you have 3/4 of a gallon of fuel left, how far can you drive?	4) You bought $5\frac{1}{2}$ yards of fabric for $6.75 per yard. How much did the fabric cost in total?
5) A recipe requires 1/4 teaspoon of salt, but you accidentally added 1/2 teaspoon. How much more did you add than required?	6) If a tank can hold $1\frac{3}{4}$ gallons of water and you already have 1/2 gallon in it, how much more water can you add?
7) You are painting a room and have painted 3/5 of it. If you complete another 1/5, what fraction of the room will be left unpainted?	8) A school's basketball team won 5/8 of its games and lost the rest. What fraction of its games did the team lose?
9) If you drink $2\frac{1}{2}$ bottles of water per day and decide to increase your water intake by 3/4 bottle per day, how much water will you drink each day now?	10) A project is divided into 4 phases. If you complete 3/4 of the first phase in one day, at the same rate, how many days will it take to complete the entire first phase?

Solutions:

1) If a recipe calls for 2/3 cup of milk and you want to double the recipe, how much milk will you need? - To double the recipe, multiply the amount of milk by 2: 2/3 cup × 2 = 4/3 cups or $1\frac{1}{3}$ cups of milk.	**2) You have a rope that is $7\frac{1}{2}$ feet long. You need to cut it into pieces that are 3/4 foot long each. How many pieces can you cut?** - To find out how many pieces you can cut, divide the total length of the rope by the length of each piece: $7\frac{1}{2}$ feet / (3/4) foot = (15/2) feet / (3/4) foot = (15/2) × (4/3) = 30/3 = 10 pieces.
3) A car's fuel efficiency is $20\frac{1}{2}$ miles per gallon. If you have 3/4 of a gallon of fuel left, how far can you drive? - Multiply the fuel efficiency by the amount of fuel left: 20 1/2 miles/gallon × 3/4 gallon = 41/2 miles × 3/4 = 123/8 miles = $15\frac{3}{8}$ miles.	**4) You bought $5\frac{1}{2}$ yards of fabric for $6.75 per yard. How much did the fabric cost in total?** - To find the total cost, multiply the amount of fabric by the cost per yard: $5\frac{1}{2}$ yards × $6.75 = 11/2 yards × $6.75 = $37.125. The total cost is $37.13 (rounded to the nearest cent).
5) A recipe requires 1/4 teaspoon of salt, but you accidentally added 1/2 teaspoon. How much more did you add than required? - Subtract the required amount of salt from what you added: 1/2 teaspoon - 1/4 teaspoon = 1/4 teaspoon. - You added 1/4 teaspoon more than required.	**6) If a tank can hold $1\frac{3}{4}$ gallons of water and you already have 1/2 gallon in it, how much more water can you add?** - Subtract the amount of water already in the tank from the total capacity: $1\frac{3}{4}$ gallons - 1/2 gallon = 7/4 gallons - 2/4 gallons = 5/4 gallons. - You can add 5/4 gallons or $1\frac{1}{4}$ gallons more water.
7) You are painting a room and have painted 3/5 of it. If you complete another 1/5, what fraction of the room will be left unpainted? - Add the fractions of the room you have painted: 3/5 + 1/5 = 4/5. - To find the fraction left unpainted, subtract this from 1: 1 - 4/5 = 1/5. - Thus, 1/5 of the room will be left unpainted.	**8) A school's basketball team won 5/8 of its games and lost the rest. What fraction of its games did the team lose?** - To find the fraction of games lost, subtract the fraction of games won from 1: 1 - 5/8 = 3/8. - The team lost 3/8 of its games.
9) If you drink $2\frac{1}{2}$ bottles of water per day and decide to increase your water intake by 3/4 bottle per day, how much water will you drink each day now?	**10) A project is divided into 4 phases. If you complete 3/4 of the first phase in one day, at the same rate, how many days will it take to complete the entire first phase?**

- Add the increase to your current intake: $2\frac{1}{2}$ bottles + 3/4 bottle = 5/2 bottles + 3/4 bottle = 10/4 bottles + 3/4 bottle = 13/4 bottles or $3\frac{1}{4}$ bottles per day.	- If 3/4 of the phase is completed in one day, then to complete 1/4 more (to finish the phase), it would take 1/3 of a day (since 3/4 is completed in 1 day, 1/4 would be completed in 1/3 of a day). - $\frac{(3/4)}{1\ day} = \frac{1}{x\ day}$ - Therefore, it will take 1 + 1/3 days to complete the entire first phase, or $1\frac{1}{3}$ days.

1-4. Absolute value and its applications

1. $5+(2^2\times2)$ 2. $(3^2-1)\div2$ 3. $4\times(2+2^2)$ 4. $16\div(2^2+4)$	13 4 24 2

1. Absolute Value:
- The absolute value of a number is the distance of the number from zero on the number line, without considering the direction.
- Notation: |x|
- Examples: |5| = 5, |-5| = 5.

2. Applications of Absolute Value:
- Temperature Changes: Absolute value can be used to calculate the total change in temperature, regardless of whether the temperature increased or decreased.
- Elevation and Depth: Absolute value helps in determining the distance above or below a reference point (like sea level).
- Distance Between Points: On a number line, the distance between two points can be found using the absolute value of their difference.

3. Solving Absolute Value Equations:
- Equations involving absolute values can have two possible solutions.
- Example: |x - a| = b implies $x-a=b$ or $x-a=-b$.

Formulas:

1. Absolute Value:

$$|x|=\begin{cases} x & \text{if } x\geq 0 \\ -x & \text{if } x<0 \end{cases}$$

2. Distance Between Two Points on a Number Line:

$$\text{Distance}=|a-b|$$

3. Solving Absolute Value Equations:

$$|x-a|=b \Rightarrow x-a=b \text{ or } x-a=-b$$

Exercise

1) What is the absolute value of -5?	2) If the temperature was 3 degrees below zero yesterday and 8 degrees above zero today, what is the total change in temperature?
3) The sea level is considered 0. If a submarine is 200 meters below sea level and then rises 75 meters, what is its new position relative to sea level?	4) Find the absolute value of the difference between -4 and 7.
5) A number line has points A and B, where A is at -3 and B is at 4. What is the distance between A and B?	6) If you owe your friend $15 (represented as -15) and you pay him back $10, how much do you still owe?
7) Solve for x in the equation $\|x - 2\| = 5$.	8) The elevation of a mountain peak is 1,450 meters above sea level, and a valley is 350 meters below sea level. What is the absolute difference in elevation between the peak and the valley?
9) A student scores -2 on a test for each question they answer incorrectly. If the student answered 12 questions wrong, what is their score? (if all correct, then the score is 100)	10) If the absolute value of a number minus 4 is 9, what are the possible values of the number?

Solutions:

1) What is the absolute value of -5? - The absolute value of -5 is 5. - Absolute value represents the distance of a number from zero on the number line, regardless of direction.	**2) If the temperature was 3 degrees below zero yesterday and 8 degrees above zero today, what is the total change in temperature?** - The total change in temperature is the absolute value of the difference: $\lvert 3 - 8 \rvert = \lvert -5 \rvert = 5$ degrees.
3) The sea level is considered 0. If a submarine is 200 meters below sea level and then rises 75 meters, what is its new position relative to sea level? - The submarine's new position is $-200 + 75 = -125$ meters. - Its position relative to sea level is 125 meters below.	**4) Find the absolute value of the difference between -4 and 7.** - The difference between -4 and 7 is $7 - (-4) = 11$. - The absolute value of 11 is 11.
5) A number line has points A and B, where A is at -3 and B is at 4. What is the distance between A and B? - The distance between A and B is the absolute value of their difference: $\lvert (-3) - 4 \rvert = \lvert -7 \rvert = 7$ units.	**6) If you owe your friend \$15 (represented as -15) and you pay him back \$10, how much do you still owe?** - After paying back \$10, you still owe \$15 - \$10 = \$5. - The absolute value of what you owe is $\lvert -\$5 \rvert = \$5$.
7) Solve for x in the equation $\lvert x - 2 \rvert = 5$. - The equation $\lvert x - 2 \rvert = 5$ means $x - 2 = 5$ or $x - 2 = -5$. - Solving each equation: - For $x - 2 = 5$, $x = 7$. - For $x - 2 = -5$, $x = -3$. - So, x can be 7 or -3.	**8) The elevation of a mountain peak is 1,450 meters above sea level, and a valley is 350 meters below sea level. What is the absolute difference in elevation between the peak and the valley?** - The absolute difference is $\lvert 1{,}450 - (-350) \rvert = 1{,}450 + 350 = 1{,}800$ meters.
9) A student scores -2 on a test for each question they answer incorrectly. If the student answered 12 questions wrong, what is their score? (if all correct, then the score is 100) - The student's score is 12 questions × -2 points/question = -24 points. - The score is $100 – 24 = 76$.	**10) If the absolute value of a number minus 4 is 9, what are the possible values of the number?** - The equation is $\lvert x - 4 \rvert = 9$. This means $x - 4 = 9$ or $x - 4 = -9$. Solving each: - For $x - 4 = 9$, $x = 13$. - For $x - 4 = -9$, $x = -5$. - The possible values of the number are 13 and -5.

1-5. Simplifying and operating with algebraic fractions

1. 2^3+6 2. $10-2^2$ 3. $3^2\times 2$ 4. $18\div 3^2$	14 6 18 2

1. Simplifying Algebraic Fractions:
- Factor the numerator and the denominator, then cancel out common factors.

2. Adding and Subtracting Algebraic Fractions:
- Find a common denominator, convert each fraction, then add or subtract the numerators.

3. Multiplying Algebraic Fractions:
- Multiply the numerators together and the denominators together, then simplify.

4. Dividing Algebraic Fractions:
- Multiply by the reciprocal of the divisor.

5. Factoring:
- Factor polynomials to simplify expressions, particularly for common factors in the numerator and denominator.

6. Solving Equations Involving Algebraic Fractions:
- Clear fractions by multiplying through by the least common denominator (LCD), then solve the resulting equation.

Formulas:

1. Simplifying Algebraic Fractions: $\frac{a^2b}{ab}=\frac{a}{1}=a$

2. Adding/Subtracting Algebraic Fractions: $\frac{a}{x}+\frac{b}{y}=\frac{ay+bx}{xy}$

3. Multiplying Algebraic Fractions: $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$

4. Dividing Algebraic Fractions: $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$

5. Factoring Quadratic Expressions: $x^2-9=(x-3)(x+3)$

6. Solving Algebraic Equations:

$$\frac{x+2}{x-3}=2 \Rightarrow x+2=2(x-3) \Rightarrow x+2=2x-6 \Rightarrow 8=x \Rightarrow x=8$$

Exercise

1) Simplify the algebraic fraction $\frac{2x^2}{4x}$.	2) Add the algebraic fractions $\frac{1}{x}$ and $\frac{2}{x+2}$.
3) Subtract $\frac{3y}{4}$ from $\frac{5y}{6}$.	4) Multiply the algebraic fractions $\frac{x}{y}$ and $\frac{2y}{3x}$.
5) Divide the algebraic fraction $\frac{4x^2}{5y}$ by $\frac{2x}{15y^2}$.	6) Simplify the algebraic expression $\frac{x^2-9}{x^2-3x+2}$ by factoring.

7) If $\frac{x}{y}+\frac{y}{x}=3$, what is $\frac{x^2+y^2}{xy}$?	8) How do you simplify $\frac{2x^3-8x}{4x}$?
9) Solve for x in the equation $\frac{x+2}{x-3}=2$.	10) If $\frac{3a-2b}{4b-2a}=\frac{3}{4}$, what is the relationship between a and b?

Solutions:

1) Simplify the algebraic fraction $\frac{2x^2}{4x}$. - To simplify $\frac{2x^2}{4x}$, divide both the numerator and the denominator by their greatest common factor, which is 2x: $\frac{2x^2}{4x}=\frac{x}{2}$.	**2) Add the algebraic fractions $\frac{1}{x}$ and $\frac{2}{x+2}$.** - To add $\frac{1}{x}$ and $\frac{2}{x+2}$, find a common denominator, which is $x(x+2)$: $\frac{1(x+2)}{x(x+2)}+\frac{2x}{x(x+2)}=\frac{x+2+2x}{x(x+2)}=\frac{3x+2}{x(x+2)}$.
3) Subtract $\frac{3y}{4}$ from $\frac{5y}{6}$. - To subtract $\frac{3y}{4}$ from $\frac{5y}{6}$, find a common denominator, which is 12: $\frac{5y}{6}-\frac{3y}{4}=\frac{10y}{12}-\frac{9y}{12}=\frac{y}{12}$.	**4) Multiply the algebraic fractions $\frac{x}{y}$ and $\frac{2y}{3x}$.** - To multiply $\frac{x}{y}$ by $\frac{2y}{3x}$, multiply the numerators and the denominators: $\frac{x\cdot 2y}{y\cdot 3x}=\frac{2xy}{3xy}$. - The xy terms cancel out, leaving $\frac{2}{3}$.

5) Divide the algebraic fraction $\frac{4x^2}{5y}$ by $\frac{2x}{15y^2}$.

- To divide $\frac{4x^2}{5y}$ by $\frac{2x}{15y^2}$, multiply the first fraction by the reciprocal of the second: $\frac{4x^2}{5y} \times \frac{15y^2}{2x} = \frac{4 \cdot 15 \cdot x^2 \cdot y^2}{5 \cdot 2 \cdot y \cdot x}$ $= \frac{60x^2y^2}{10xy} = \frac{6xy}{1} = 6xy$.

6) Simplify the algebraic expression $\frac{x^2-9}{x^2-3x+2}$ by factoring.

- First, factor both the numerator and the denominator: x^2-9 can be factored as $(x+3)(x-3)$, and x^2-3x+2 can be factored as $(x-1)(x-2)$.
- Thus, $\frac{x^2-9}{x^2-3x+2} = \frac{(x+3)(x-3)}{(x-1)(x-2)}$.
- Without common factors, this is the simplified form.

7) If $\frac{x}{y} + \frac{y}{x} = 3$, what is $\frac{x^2+y^2}{xy}$?

- To find $\frac{x^2+y^2}{xy}$, note that it resembles the given equation but squared.
- Since $\frac{x}{y} + \frac{y}{x} \Rightarrow \frac{x^2+y^2}{xy}$, the answer is $\frac{x^2+y^2}{xy} = 3$.

8) How do you simplify $\frac{2x^3-8x}{4x}$?

- Factor out the greatest common factor in the numerator: $2x^3 - 8x = 2x(x^2-4)$.
- Then simplify the fraction: $\frac{2x(x^2-4)}{4x} = \frac{x^2-4}{2} = \frac{x^2-2^2}{2}$, which is $\frac{x^2-4}{2}$ after factoring.

9) Solve for x in the equation $\frac{x+2}{x-3} = 2$.

- Cross-multiply to solve for x: $(x+2) = 2(x-3)$, which simplifies to $x+2 = 2x-6$.
- Subtract x from both sides to get $2 = x-6$, then add 6 to both sides to find $x = 8$.

10) If $\frac{3a-2b}{4b-2a} = \frac{3}{4}$, what is the relationship between a and b?

- Cross-multiply to find the relationship: $3a - 2b = \frac{3}{4}(4b-2a)$, which simplifies to $3a - 2b = 3b - \frac{3}{2}a$.
- Rearranging the terms to one side gives $3a + \frac{3}{2}a = 2b + 3b$, simplifying further to $4\frac{1}{2}a = 5b$, which gives the relationship $a = \frac{5}{4.5}b = \frac{10}{9}b$, indicating that a is $\frac{10}{9}$ times b. ($a = \frac{10}{9}b$)

1-6. Conversion between mixed numbers and improper fractions

1. $(2+3)\times 2^2$ 2. $(5-2)^2+3$ 3. $3^3 \div 3$ 4. $(4\times 2)-2^3$	20 12 9 0

1. Mixed Numbers:

- A mixed number is a number that consists of an integer and a proper fraction.
- Example: $2\frac{3}{4}$

2. Improper Fractions:

- An improper fraction is a fraction where the numerator is greater than or equal to the denominator.
- Example: $\frac{9}{4}$

3. Conversion from Mixed Numbers to Improper Fractions:

- To convert a mixed number to an improper fraction, multiply the whole number part by the denominator of the fraction, add the numerator, and place the result over the original denominator.
- Formula:

$$a\frac{b}{c}=\frac{ac+b}{c}$$

4. Conversion from Improper Fractions to Mixed Numbers:

- To convert an improper fraction to a mixed number, divide the numerator by the denominator to get the integer part, and the remainder becomes the numerator of the proper fraction part.
- Formula:

$$\frac{a}{b}=q\frac{r}{b} \quad \text{where } q=\left\lfloor \frac{a}{b} \right\rfloor \text{ and } r=a \bmod b$$

Exercise

1) Convert the mixed number $2\frac{3}{4}$ to an improper fraction.	2) Convert the improper fraction 9/2 to a mixed number.
3) Change the mixed number $5\frac{1}{5}$ into an improper fraction.	4) Convert the improper fraction 15/4 into a mixed number.
5) Transform the mixed number $7\frac{2}{3}$ into an improper fraction.	6) Convert the improper fraction 22/7 into a mixed number.
7) Change the mixed number $11\frac{1}{2}$ into an improper fraction.	8) Convert the improper fraction 50/9 into a mixed number.
9) Transform the mixed number $3\frac{3}{8}$ into an improper fraction.	10) Convert the improper fraction 81/10 into a mixed number.

Solutions:

1) Convert the mixed number $2\frac{3}{4}$ to an improper fraction. - To convert 2 3/4 to an improper fraction, multiply the whole number part by the denominator of the fraction part and add the numerator: $2\times4+3=8+3=11$. - So, the improper fraction is $11/4$.	**2) Convert the improper fraction 9/2 to a mixed number.** - To convert 9/2 into a mixed number, divide the numerator by the denominator: $9\div2=4$ with a remainder of 1. - So, the mixed number is $4\frac{1}{2}$.
3) Change the mixed number $5\frac{1}{5}$ into an improper fraction. - Multiply the whole number by the denominator and add the numerator: $5\times5+1=25+1=26$. - The improper fraction is $26/5$.	**4) Convert the improper fraction 15/4 into a mixed number.** - Divide the numerator by the denominator: $15\div4=3$ with a remainder of 3. - So, the mixed number is $3\frac{3}{4}$.
5) Transform the mixed number $7\frac{2}{3}$ into an improper fraction. - Multiply the whole number by the denominator and add the numerator: $7\times3+2=21+2=23$. - The improper fraction is $23/3$.	**6) Convert the improper fraction 22/7 into a mixed number.** - Divide the numerator by the denominator: $22\div7=3$ with a remainder of 1. - So, the mixed number is $3\frac{1}{7}$.
7) Change the mixed number $11\frac{1}{2}$ into an improper fraction. - Multiply the whole number by the denominator and add the numerator: $11\times2+1=22+1=23$. - The improper fraction is $23/2$.	**8) Convert the improper fraction 50/9 into a mixed number.** - Divide the numerator by the denominator: $50\div9=5$ with a remainder of 5. - So, the mixed number is $5\frac{5}{9}$.
9) Transform the mixed number $3\frac{3}{8}$ into an improper fraction. - Multiply the whole number by the denominator and add the numerator: $3\times8+3=24+3=27$. - The improper fraction is $27/8$.	**10) Convert the improper fraction 81/10 into a mixed number.** - Divide the numerator by the denominator: $81\div10=8$ with a remainder of 1. - So, the mixed number is $8\frac{1}{10}$.

1-7. Operations with fractions and mixed numbers

1. $2\times(3+5)-4$ 2. $(4^2-2^2)\div 2$ 3. $6\div 2+3^2$ 4. $5\times(2+4)-3^2$	12 6 12 21

1. Adding Fractions and Mixed Numbers:

- Convert mixed numbers to improper fractions, find a common denominator, add the fractions, and if necessary, convert back to a mixed number.

2. Subtracting Fractions and Mixed Numbers:

- Similar to addition, convert to improper fractions, find a common denominator, subtract, and convert back if needed.

3. Multiplying Fractions and Mixed Numbers:

- Convert mixed numbers to improper fractions, multiply the numerators and denominators, simplify, and convert back if needed.

4. Dividing Fractions and Mixed Numbers:

- Convert mixed numbers to improper fractions, multiply by the reciprocal of the divisor, simplify, and convert back if needed.

Formulas:

1. Adding Fractions: $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$

2. Subtracting Fractions: $\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$

3. Multiplying Fractions: $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$

4. Dividing Fractions: $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$

Exercise

1) Add the mixed numbers $2\frac{1}{3}$ and $1\frac{2}{5}$.	2) Subtract $3\frac{4}{7}$ from $5\frac{2}{5}$. 8.
3) Multiply the mixed numbers $2\frac{1}{2}$ and $3\frac{1}{3}$. 9.	4) Divide $4\frac{3}{5}$ by $2\frac{1}{4}$. 10.
5) Simplify the expression by adding $1\frac{1}{2}$ and subtracting $2\frac{2}{3}$ from it.	6) If a cake recipe requires $2\frac{3}{4}$ cups of flour and you want to make only half of the cake, how much flour do you need?
7) A recipe calls for $3\frac{1}{2}$ cups of sugar, but you accidentally pour in 4 cups. How much sugar should you remove?	8) You ran $5\frac{1}{3}$ miles on Monday and $3\frac{2}{5}$ miles on Tuesday. How many total miles did you run?
9) Subtract $7\frac{5}{6}$ from 10.	10) If you drink $1\frac{3}{4}$ liters of water each day and decide to increase your intake by $\frac{1}{2}$ liter, how much water will you drink now?

Solutions:

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1) Add the mixed numbers $2\frac{1}{3}$ and $1\frac{2}{5}$.

- First, convert the mixed numbers to improper fractions: $2\frac{1}{3}=\frac{7}{3}$ and $1\frac{2}{5}=\frac{7}{5}$.
- Then, find a common denominator, which is 15, and add the fractions: $\frac{7}{3}\times\frac{5}{5}+\frac{7}{5}\times\frac{3}{3}=\frac{35}{15}+\frac{21}{15}=\frac{56}{15}$.
- Convert back to a mixed number: $3\frac{11}{15}$.

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2) Subtract $3\frac{4}{7}$ from $5\frac{2}{5}$.

- Convert to improper fractions: $3\frac{4}{7}=\frac{25}{7}$ and $5\frac{2}{5}=\frac{27}{5}$.
- Find a common denominator, which is 35, and subtract: $\frac{27}{5}\times\frac{7}{7}-\frac{25}{7}\times\frac{5}{5}=\frac{189}{35}-\frac{125}{35}=\frac{64}{35}$.
- Convert back to a mixed number: $1\frac{29}{35}$.

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3) Multiply the mixed numbers $2\frac{1}{2}$ and $3\frac{1}{3}$.

- Convert to improper fractions: $2\frac{1}{2}=\frac{5}{2}$ and $3\frac{1}{3}=\frac{10}{3}$.
- Multiply the fractions: $\frac{5}{2}\times\frac{10}{3}=\frac{50}{6}$.
- Simplify: $\frac{25}{3}$, which converts back to a mixed number as $8\frac{1}{3}$.

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4) Divide $4\frac{3}{5}$ by $2\frac{1}{4}$.

- Convert to improper fractions: $4\frac{3}{5}=\frac{23}{5}$ and $2\frac{1}{4}=\frac{9}{4}$.
- Divide the first fraction by the second by multiplying by the reciprocal of the second fraction: $\frac{23}{5}\times\frac{4}{9}=\frac{92}{45}$.
- Convert back to a mixed number: $2\frac{2}{45}$.

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5) Simplify the expression by adding $1\frac{1}{2}$ and subtracting $2\frac{2}{3}$ from it.

- Convert to improper fractions: $1\frac{1}{2}=\frac{3}{2}$ and $2\frac{2}{3}=\frac{8}{3}$.
- Perform the operations: $\frac{3}{2}-\frac{8}{3}$.
- Find a common denominator, which is 6, and simplify: $\frac{9}{6}-\frac{16}{6}=-\frac{7}{6}$.
- Convert to a mixed number: $-1\frac{1}{6}$.

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6) If a cake recipe requires $2\frac{3}{4}$ cups of flour and you want to make only half of the cake, how much flour do you need?

- Convert to an improper fraction: $2\frac{3}{4}=\frac{11}{4}$ cups.
- Since you're making half the cake, divide by 2: $\frac{11}{4}\div 2=\frac{11}{4}\times\frac{1}{2}=\frac{11}{8}$ cups.
- Convert back to a mixed number: $1\frac{3}{8}$ cups.

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7) **A recipe calls for $3\frac{1}{2}$ cups of sugar, but you accidentally pour in 4 cups. How much sugar should you remove?** - Convert to improper fractions: $3\frac{1}{2}=\frac{7}{2}$ and $4=\frac{8}{2}$. - Subtract to find the excess: $\frac{8}{2}-\frac{7}{2}=\frac{1}{2}$ cup. - You should remove 1/2 cup of sugar.	**8) You ran $5\frac{1}{3}$ miles on Monday and $3\frac{2}{5}$ miles on Tuesday. How many total miles did you run?** - Convert to improper fractions: $5\frac{1}{3}=\frac{16}{3}$ and $3\frac{2}{5}=\frac{17}{5}$. - Find a common denominator, which is 15, and add: $\frac{16}{3}\times\frac{5}{5}+\frac{17}{5}\times\frac{3}{3}=\frac{80}{15}+\frac{51}{15}=\frac{131}{15}$. - Convert back to a mixed number: $8\frac{11}{15}$ miles.
9) Subtract $7\frac{5}{6}$ from 10. - Convert 7 5/6 to an improper fraction: $7\frac{5}{6}=\frac{47}{6}$. - Since 10 is a whole number, convert it to $\frac{60}{6}$ to use the same denominator. - Subtract: $\frac{60}{6}-\frac{47}{6}=\frac{13}{6}$. - Convert back to a mixed number: $2\frac{1}{6}$.	**10) If you drink $1\frac{3}{4}$ liters of water each day and decide to increase your intake by $\frac{1}{2}$ liter, how much water will you drink now?** - Convert 1 3/4 to an improper fraction: $1\frac{3}{4}=\frac{7}{4}$ liters. - Add 1/2 liter: $\frac{7}{4}+\frac{1}{2}=\frac{7}{4}+\frac{2}{4}=\frac{9}{4}$ liters. - Convert back to a mixed number: $2\frac{1}{4}$ liters.

1-8. Fraction to decimal to percent conversion

1. $(3^2 - 1) + (-2)$ 2. $4 \times (-2 + 3^2)$ 3. $(-3 + 5) \times 2^2$ 4. $7 - (2^3 + -5)$	6 28 8 4

1. Converting Fractions to Decimals:
- Divide the numerator by the denominator.
- Example: $\frac{1}{2}$ = 0.5

2. Converting Decimals to Percents:
- Multiply the decimal by 100 and add a percent sign.
- Example: 0.5 × 100 = 50%

3. Converting Fractions Directly to Percents:
- Convert the fraction to a decimal, then to a percent, or multiply the fraction by 100%.
- Example: $\frac{1}{2}$ × 100% = 50%

Exercise

1) Convert the fraction 1/2 to a decimal and then to a percent.	2) Change the fraction 3/4 to a decimal and then to a percent.
3) Convert the fraction 1/5 to a decimal and then to a percent.	4) Change the fraction 2/3 to a decimal and then to a percent.

5) Convert the fraction 4/5 to a decimal and then to a percent.	6) Change the fraction 7/8 to a decimal and then to a percent.
7) Convert the fraction 5/16 to a decimal and then to a percent.	8) Change the fraction 3/10 to a decimal and then to a percent.
9) Convert the fraction 9/20 to a decimal and then to a percent.	10) Change the fraction 15/100 to a decimal and then to a percent.

Solutions:

1) Convert the fraction 1/2 to a decimal and then to a percent. - To convert 1/2 to a decimal, divide the numerator by the denominator: $1 \div 2 = 0.5$. - To convert 0.5 to a percent, multiply by 100: $0.5 \times 100 = 50\%$.	**2) Change the fraction 3/4 to a decimal and then to a percent.** - Divide 3 by 4 to get the decimal: $3 \div 4 = 0.75$ - Then, convert 0.75 to a percent by multiplying by 100: $0.75 \times 100 = 75\%$.
3) Convert the fraction 1/5 to a decimal and then to a percent. - To convert 1/5 to a decimal, divide 1 by 5: $1 \div 5 = 0.2$. - To convert 0.2 to a percent, multiply by 100: $0.2 \times 100 = 20\%$.	**4) Change the fraction 2/3 to a decimal and then to a percent.** - Divide 2 by 3 to get the decimal: $2 \div 3 = 0.666...$. - Round to two decimal places for simplicity: 0.67. - Then, convert 0.67 to a percent by multiplying by 100: $0.67 \times 100 = 67\%$.
5) Convert the fraction 4/5 to a decimal and then to a percent. - To convert 4/5 to a decimal, divide 4 by 5: $4 \div 5 = 0.8$. - To convert 0.8 to a percent, multiply by 100: $0.8 \times 100 = 80\%$.	**6) Change the fraction 7/8 to a decimal and then to a percent.** - Divide 7 by 8 to get the decimal: $7 \div 8 = 0.875$. - Then, convert 0.875 to a percent by multiplying by 100: $0.875 \times 100 = 87.5\%$.
7) Convert the fraction 5/16 to a decimal and then to a percent. - To convert 5/16 to a decimal, divide 5 by 16: $5 \div 16 = 0.3125$. - To convert 0.3125 to a percent, multiply by 100: $0.3125 \times 100 = 31.25\%$.	**8) Change the fraction 3/10 to a decimal and then to a percent.** - Divide 3 by 10 to get the decimal: $3 \div 10 = 0.3$. - Then, convert 0.3 to a percent by multiplying by 100: $0.3 \times 100 = 30\%$.
9) Convert the fraction 9/20 to a decimal and then to a percent. - To convert 9/20 to a decimal, divide 9 by 20: $9 \div 20 = 0.45$. - To convert 0.45 to a percent, multiply by 100: $0.45 \times 100 = 45\%$.	**10) Change the fraction 15/100 to a decimal and then to a percent.** - To convert 15/100 to a decimal, simply move the decimal point two places to the left: 0.15. - To convert 0.15 to a percent, multiply by 100: $0.15 \times 100 = 15\%$.

1-9. Understanding and using exponents and square roots

1. -2^3+5 2. $3\times(4-7)+2^2$ 3. $(5^2-9)\div(-2)$ 4. $-3+6^2\div3$	1. -3 2. -5 3. -8 4. 9

1. Exponents:

- An exponent indicates how many times a number, called the base, is multiplied by itself.
- Example: $2^3=2\times2\times2=8$

2. Square Roots:

- The square root of a number is a value that, when multiplied by itself, gives the original number.
- Example: $\sqrt{64}=8$

3. Properties of Exponents:

- $a^m\times a^n=a^{m+n}$
- $(a^m)^n=a^{mn}$
- $a^0=1$ (for any non-zero a)

4. Properties of Square Roots:

- $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$

Exercise

1) What is the value of 2^3?	2) Simplify the expression $4^2 \times 4^3$.
3) What is the square root of 64?	4) Simplify the expression $\sqrt{49} \times \sqrt{9}$.
5) What is 5^0?	6) If $x^2 = 81$, what are the possible values of x?
7) Simplify the expression $(2^3)^2$.	8) What is the cube root of 27?
9) Simplify the expression $\sqrt{25} + \sqrt{16}$.	10) If $4x^2 = 100$, what are the possible values of x?

Solutions:

1) What is the value of 2^3 ? - The value of 2^3 is $2\times2\times2=8$.	**2) Simplify the expression $4^2\times4^3$.** - To simplify $4^2\times4^3$, add the exponents: $4^{2+3}=4^5=1024$.
3) What is the square root of 64? - The square root of 64 is 8, because $8\times8=64$. - Answer: 8	**4) Simplify the expression $\sqrt{49}\times\sqrt{9}$.** - $\sqrt{49}\times\sqrt{9}=7\times3=21$.
5) What is 5^0 ? - Any number raised to the power of 0 is 1. - Therefore, $5^0=1$.	**6) If $x^2=81$, what are the possible values of x ?** - The possible values of x are 9 and -9, because $9^2=81$ and $(-9)^2=81$. - Answers are 9 and -9
7) Simplify the expression $(2^3)^2$. - To simplify $(2^3)^2$, multiply the exponents: $2^{3\times2}=2^6=64$.	**8) What is the cube root of 27?** - The cube root of 27 is 3, because $3^3=27$.' - $\sqrt[3]{27}=3$
9) Simplify the expression $\sqrt{25}+\sqrt{16}$. - $\sqrt{25}+\sqrt{16}=5+4=9$.	**10) If $4x^2=100$, what are the possible values of x ?** - To find x, divide both sides by 4: $x^2=25$. - The possible values for x are 5 and -5, because $5^2=25$ and $(-5)^2=25$. - Answers are 5 and -5

1-10. Application of scientific notation

1. $(3+7)\div 2-3^2$ 2. $-2\times(3-2^3)$ 3. $4^2-(3+5)\times 2$ 4. $7-(3\div -1+2^2)$	-4 10 0 6

1. Scientific Notation:

- A way to express very large or very small numbers.
- Form: $a\times 10^n$ where $1\le a<10$ and n is an integer.
- Example: 4500000 = 4.5×10^6 .

2. Converting to Scientific Notation:

- Move the decimal point so that there is one non-zero digit to its left.
- Count the number of places the decimal point has moved; this becomes the exponent.
- If the original number is greater than 1, the exponent is positive. If less than 1, the exponent is negative.

3. Operations with Scientific Notation:

- Multiplication: Multiply the coefficients and add the exponents.

$$(a\times 10^m)\times(b\times 10^n)=(a\times b)\times 10^{m+n}$$

- Division: Divide the coefficients and subtract the exponents.

$$\frac{a\times 10^m}{b\times 10^n}=\frac{a}{b}\times 10^{m-n}$$

Exercise

1) Express 0.00056 in scientific notation.	2) Write 4500000 in scientific notation.

3) Convert 3.2×10^3 to standard form.	4) Multiply 2×10^4 by 3×10^3.
5) Divide 5×10^6 by 2.5×10^2.	6) Simplify $(4\times10^{-3})\times(2\times10^2)$.
7) What is the sum of 6×10^3 and 4×10^4 ?	8) If a cell has a diameter of 2×10^{-6} meters, express this diameter in micrometers ($1\text{ micrometer}=10^{-6}\text{ meters}$).
9) Simplify $(5\times10^8)/(1\times10^4)$.	10) If the Earth is approximately 1.5×10^8 kilometers from the sun, express this distance in meters.

Solutions:

1) Express 0.00056 in scientific notation. - To express 0.00056 in scientific notation, move the decimal point 4 places to the right: 5.6×10^{-4}.	**2) Write 4500000 in scientific notation.** - To write 4500000 in scientific notation, move the decimal point 6 places to the left: 4.5×10^{6}.
3) Convert 3.2×10^{3} to standard form. - To convert 3.2×10^{3} to standard form, move the decimal point 3 places to the right: 3200.	**4) Multiply 2×10^{4} by 3×10^{3}.** - To multiply, multiply the coefficients and add the exponents: $2\times3\times10^{4+3}=6\times10^{7}$.
5) Divide 5×10^{6} by 2.5×10^{2}. - To divide, divide the coefficients and subtract the exponents: $\frac{5}{2.5}\times10^{6-2}=2\times10^{4}$.	**6) Simplify $(4\times10^{-3})\times(2\times10^{2})$.** - To simplify, multiply the coefficients and add the exponents: $4\times2\times10^{-3+2}=8\times10^{-1}$.
7) What is the sum of 6×10^{3} and 4×10^{4}? - To add, express both numbers with the same exponent: $6\times10^{3}+40\times10^{3}=46\times10^{3}$ $=4.6\times10^{4}$.	**8) If a cell has a diameter of 2×10^{-6} meters, express this diameter in micrometers ($1\text{ micrometer}=10^{-6}\text{ meters}$).** - To express the diameter in micrometers, use the conversion factor: $2\times10^{-6}\text{ meters}=2\text{ micrometers}$.
9) Simplify $(5\times10^{8})/(1\times10^{4})$. - To simplify, divide the coefficients and subtract the exponents: $5/1\times10^{8-4}=5\times10^{4}$.	**10) If the Earth is approximately 1.5×10^{8} kilometers from the sun, express this distance in meters.** - Knowing that 1 kilometer equals 10^{3} meters, convert the distance to meters: $1.5\times10^{8}\times10^{3}=1.5\times10^{11}$ meters.

1-11. Comparing and ordering rational numbers

1. $\sqrt{16}+3$ 2. $2^2 \times \sqrt{9}$ 3. $3^2 - \sqrt{25}$ 4. $\sqrt{36} \div 2 + 2$	7 12 4 5

1. Comparing Rational Numbers:

- To compare rational numbers, convert them to a common form (fractions, decimals, or percentages).
- Compare the converted forms to determine which is greater or smaller.

2. Ordering Rational Numbers:

- To order rational numbers, convert them to a common form and then arrange them in ascending or descending order.

Exercise

1) Arrange the following numbers in ascending order: 1/2, 3/4, 2/3.	2) Which is greater, -3/4 or -1/2?
3) Order the following numbers from least to greatest: -2, 3/4, -1/3, 0.	4) Compare 5/8 and 0.6. Which is larger?
5) Arrange -1/2, -2/3, and -3/4 in descending order.	6) Which is smaller, 0.25 or 1/5?

7) Order these numbers from greatest to least: 1.2, 7/6, $1\frac{1}{3}$, 1.15.	8) Which is greater, 2.5 or 5/2?
9) Arrange the following in ascending order: -3.5, -7/2, 3/2, -1.5.	10) Compare -1/4 and -0.25. Which is larger?

Solutions:

1) Arrange the following numbers in ascending order: 1/2, 3/4, 2/3. - Convert each fraction to a decimal or find a common denominator to compare: 1/2 = 0.5, 3/4 = 0.75, 2/3 ≈ 0.67. - So, in ascending order: 1/2, 2/3, 3/4.	**2) Which is greater, -3/4 or -1/2?** - Convert each fraction to a decimal to compare: -3/4 = -0.75, -1/2 = -0.5. - Since -0.5 is closer to zero, -1/2 is greater than -3/4.
3) Order the following numbers from least to greatest: -2, 3/4, -1/3, 0. - In ascending order: -2, -1/3, 0, 3/4.	**4) Compare 5/8 and 0.6. Which is larger?** - Convert 5/8 to a decimal: 5/8 = 0.625. - Since 0.625 > 0.6, 5/8 is larger than 0.6.
5) Arrange -1/2, -2/3, and -3/4 in descending order. - Convert each fraction to a decimal to compare: -1/2 = -0.5, -2/3 ≈ -0.67, -3/4 = -0.75. - In descending order: -1/2, -2/3, -3/4.	**6) Which is smaller, 0.25 or 1/5?** - Convert 1/5 to a decimal: 1/5 = 0.2. - Since 0.2 < 0.25, 1/5 is smaller than 0.25.
7) Order these numbers from greatest to least: 1.2, 7/6, $1\frac{1}{3}$, 1.15. - Convert fractions to decimals: 7/6 = 1.17, $1\frac{1}{3}$ = 4/3 ≈ 1.33. - So, in descending order: $1\frac{1}{3}$, 1.2, 7/6, 1.15.	**8) Which is greater, 2.5 or 5/2?** - Since 5/2 equals 2.5, they are equal.
9) Arrange the following in ascending order: -3.5, -7/2, 3/2, -1.5. - Recognize that -7/2 equals -3.5. - So, in ascending order: -3.5 (-7/2), -1.5, 3/2.	**10) Compare -1/4 and -0.25. Which is larger?** - Since -1/4 equals -0.25, they are equal.

1-12. Recognizing and representing proportional relationships between quantities

1. $\sqrt{49}-2^3$ 2. $2\times\sqrt{16}+3$ 3. $3^2+\sqrt{64}$ 4. $\sqrt{81}\div 3-2$	1. -1 2. 14 3. 17 4. 1

1. Proportional Relationships:

- Two quantities are proportional if they maintain a constant ratio.
- If a and b are proportional, then $\frac{a}{b}=k$ where k is the constant of proportionality.

2. Solving Proportional Problems:

- Set up a ratio or fraction equal to the constant ratio and solve for the unknown.

Exercise

1) If a recipe calls for 2 cups of flour for every 3 cups of sugar, how many cups of flour are needed for 9 cups of sugar?	2) A car travels 150 miles in 3 hours. At the same rate, how far will it travel in 5 hours?
3) If 4 pens cost $2, how much do 10 pens cost?	4) A map scale shows that 1 inch equals 100 miles. How many miles are represented by 8 inches on the map?

5) In a school, the ratio of boys to girls is 3:5. If there are 120 girls, how many boys are there?	6) A recipe requires 5 grams of spice for every 2 kilograms of meat. How much spice is needed for 7 kilograms of meat?
7) If a car uses 8 liters of gasoline to travel 100 kilometers, how much gasoline is needed to travel 250 kilometers?	8) A printer prints 200 pages in 4 minutes. At this rate, how many pages can it print in 10 minutes?
9) The ratio of water to cement in a mixture is 2:3. How much cement is needed if 40 liters of water are used?	10) A copy machine makes 75 copies per minute. How long does it take to make 600 copies?

Solutions:

1) If a recipe calls for 2 cups of flour for every 3 cups of sugar, how many cups of flour are needed for 9 cups of sugar? - Use the proportion $2/3 = x/9$. - Solving for x, $x=(2\times 9)/3=18/3=6$. - You need 6 cups of flour for 9 cups of sugar.	**2) A car travels 150 miles in 3 hours. At the same rate, how far will it travel in 5 hours?** - The rate is $150 \text{ miles}/3 \text{ hours} = x \text{ miles/5 hour}$. - $150 \times 5 = 3 \times x$ - For 5 hours, distance is $\frac{150\times 5}{3}=250$ miles.
3) If 4 pens cost $2, how much do 10 pens cost? - Use the proportion $4 \text{ pens}/\$2 = 10 \text{ pens}/x$. - Solving for x, $x=(10\times 2)/4=20/4=\$5$. - 10 pens cost $5.	**4) A map scale shows that 1 inch equals 100 miles. How many miles are represented by 8 inches on the map?** - If 1 inch represents 100 miles, then 8 inches represent $8\times 100=800$ miles.
5) In a school, the ratio of boys to girls is 3:5. If there are 120 girls, how many boys are there? - Use the ratio $3 \text{ boys} : 5 \text{ girls} = x \text{ boys} : 120 \text{ girls}$. - Solving for x, $x=(3\times 120)/5=360/5=72$. - There are 72 boys.	**6) A recipe requires 5 grams of spice for every 2 kilograms of meat. How much spice is needed for 7 kilograms of meat?** - Use the proportion $5 \text{ grams}/2 \text{ kg} = x \text{ grams}/7 \text{ kg}$. - Solving for x, $x=(5\times 7)/2=35/2=17.5$ grams. - You need 17.5 grams of spice for 7 kilograms of meat.
7) If a car uses 8 liters of gasoline to travel 100 kilometers, how much gasoline is needed to travel 250 kilometers? - Use the proportion $8 \text{ liters}/100 \text{ km} = x \text{ liters}/250 \text{ km}$. - Solving for x, $x=(8\times 250)/100=2000/100=20$ liters. - You need 20 liters of gasoline to travel 250 kilometers.	**8) A printer prints 200 pages in 4 minutes. At this rate, how many pages can it print in 10 minutes?** - The rate is $200 \text{ pages}/4 \text{ minutes} = 50 \text{ pages/minute}$. - For 10 minutes, pages = rate × time = $50\times 10=500$ pages.
9) The ratio of water to cement in a mixture is 2:3. How much cement is needed if 40 liters of water are used? - Use the ratio $2 \text{ liters water} : 3 \text{ liters cement} = 40 : x$. - Solving for x, $x=(3\times 40)/2=120/2=60$ liters. You need 60 liters of cement.	**10) A copy machine makes 75 copies per minute. How long does it take to make 600 copies?** - Use the rate to find time: $\frac{600 \text{ copies}}{\left(75\frac{\text{copies}}{\text{minute}}\right)}=8$ minutes. - It takes 8 minutes to make 600 copies.